Methods are presented for calculating distributions of tangential velocity and static pressure and the lengthwise and radial heat transfer in the axial zone of a cyclonic flow.

Most schemes and methods presently available for the aerodynamic design of cyclone and eddy devices, such as [1-3], presume the independence of the distribution of the tangential component of flow velocity on the longitudinal coordinate $x$. Attempts at numerical analysis of twisted flows in the axial region on the basis of the general equations of motion [4, 5] have not yet backed up the empirically established substantial deformation of the profile of $w_{\varphi}$, particularly with large values of $\bar{d}_{\text {out }}$ and in sections near the outlet. This deformation is connected mainly with the effect of axial back flow. The available recommendations oncalculating heat transfer in the axial zone of cyclone chambers, analyzed in [6], make it possible to evaluate only values of $\alpha$ averaged over the length of the heat-exchange surface; in several cases, the theoretical formulas are inconvenient to use or do not have the necessary generality. Quite often this has to do with the fact that in forming the governing similitude numbers insufficient account is taken of flow features in the zone being analyzed.

In solving dynamic and thermal problems, we use a cylindrical coordinate system with its origin in the plane of the outlet hole of a cyclone chamber (Fig. 1). The cyclonic flow in the core of the flow will be assumed to be axisymmetric.

We assign the distribution of $W_{\varphi}$ with respect to $r$ by an approximation having the form

$$
\begin{equation*}
\bar{\omega}=\left(\frac{2 \eta}{1+\eta^{x}}\right)^{n}, \tag{1}
\end{equation*}
$$

where $n$ and $x$ are constants.
With $n=1$ and $x=2$, Eq. (I) corresponds to the familiar Vulis-Ustimenko approximation [1]. (A similar form of the relation $\bar{w}=\bar{w}(n)$ was established in studying rotational fluid motion in vortices [7, 8].) In the scheme in [2] for calculating the aerodynamics of unloaded


Fig. 1. Diagram of the cyclone chamber.

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Fig. 2. Radial distribution of $\bar{\omega}_{x}, \bar{w}, \bar{u}$, and $\bar{P}$ in the axial zone of a cyclone flow: a) solid line - calculation by Eqs. (1) and (3); dashed line - calculation by the method in [19]; test data: 1) $[13]$;2) $[14]$; 3) $[15]$; 4) $[16]$;5) $[9]$;6) [17]; 7) [18]; 8-12) author; b) calculation: 1) by Eq. (1) ; 2) (15); 3) (16).
cyclone chambers, the exponent n is determined from the condition of maximum circulation $\Gamma=$ w ) on the boundary of the flow core $\mathrm{n}_{\mathrm{c}}$. Henceforth, a similar approximation, with allowance for the change to a new variable $n^{*}=\left(r-r_{g}\right) /\left(R_{\varphi m}-r_{g}\right)$ was used to develop a method for the aerodynamic design of cyclone-vortex heaters [6].

Analysis of the empirical distributions of $w_{\infty}$ with respect to the radius shows that they converge satisfactorily with the results calculated by Eq. (1) with allowance for the abovenoted method of determining $n$ only in the "quasipotential" flow region ( $1 \leqslant n \leqslant n_{c}$ ). The calculation for the zone of "quasisolid" rotation is unsubstantiated and agrees poorly with the empirical data, especially over a large radial expanse up to $r_{c}=(0.7-0.9) R$. The need to separately determine the exponent $n$ for each zone was also indicated in [9, 10].

The study [11] proposed a method of determining $n$ to calculate $w_{\varphi}$ in the axial region on the basis of the condition of the existence of a maximum of the angular velocity $\bar{\omega}_{x}$ reached on the radius $0 \leqslant n_{\omega} \leqslant 1$ (Fig. 1). With

$$
\begin{align*}
\frac{\partial \bar{\omega}_{x}}{\partial \eta} & =\frac{\partial}{\partial \eta}\left[\frac{1}{\eta}\left(\frac{2 \eta}{1+\eta^{x}}\right)^{n}\right]=0  \tag{2}\\
n & =n_{\mathrm{\omega}}=\frac{1+\eta_{\omega}^{x}}{1-(x-1) \eta_{\omega}^{x}} . \tag{3}
\end{align*}
$$

Equation (2) actually corresponds to the condition for stable rotation of a gas [12] in the axial region of a cyclonic flow, while the radius $n_{\omega}$ is the boundary between the zones with a conservative $\left(0 \leqslant n<\eta_{\omega}\right)$ and active ( $n>n_{\omega}$ ) character of the effect of inertial body forces on the flow.

To improve the convergence of the calculation with the experiment, including with limiting values $\omega=0,1$, we propose to assign the value $x=1.87$ instead of 2 .

Figure 2 shows graphs of the radial distribution of $\bar{\omega}_{\mathrm{x}}$ in unloaded chambers calculated with allowance for Eqs. (1) and (3) in the range of $n_{\omega}$ from 0 to 1 . The figure also compares calculated curves of $\bar{w}_{X}$ and $\bar{w}$ with our empirical data and data from several other studies [9, 13-18]. It can be seen from the figure that the calculated results agree well with the empirical results.

It was possible to establish the dependence of the characteristic radius $\eta_{\omega}$ on $\overline{\mathrm{f}}_{\text {in }}$, $\overline{\mathrm{d}}_{\text {out }}, \overline{\mathrm{L}}$, and $\overline{\mathrm{x}}$ from preliminary analysis of numerous empirical distributions of tangential velocity with different (including zero [19]) degrees of filling (loading) of the axial zone of a cyclone chamber with cylindrical inserts. An analytic relation connecting $\eta_{\omega}$ with the main geometric parameters of the cyclone chamber and the longitudinal coordinate can be obtained by using a spiral flow model in the axial region [20].

Following [21], we write the vector equation of the mean motion of a viscous incompressible turbulent flow in the Helmholtz form, i.e., in the vorticity transport form:

$$
\begin{equation*}
\frac{\partial\langle\boldsymbol{\omega}\rangle}{\partial t}=\nabla \times(\langle\mathbf{V}\rangle \times\langle\boldsymbol{\omega}\rangle)+\nabla \times\left(\mathbf{V}^{\prime} \times \boldsymbol{\omega}^{\prime}\right)+v \nabla^{2}\langle\omega\rangle \tag{4}
\end{equation*}
$$

For steady spiral flow $\frac{\partial\langle\boldsymbol{\omega}\rangle}{\partial t}=0,\langle\mathbf{V}\rangle \times\langle\boldsymbol{\omega}\rangle=0$. Considering the relationship between the pulsative and mean components of angular velocity $\omega^{\prime}=\eta(\partial\langle\omega\rangle / \partial r$ ) ( $Z$ is the turbulence scale) in accordance with the vorticity transport theory of $G$. Taylor and condition (2) at $r=r_{\omega}$, we have

$$
\boldsymbol{\omega}_{m}^{\prime}=0 \text { and }\left(\mathbf{V}^{\prime} \times \boldsymbol{\omega}^{\prime}\right)_{m}=0
$$

Thus, Eq. (4), written relative to $\bar{\omega}_{\mathrm{xm}}$ (we omit the averaging symbols) and represented in dimensionless form, is simplified and becomes an elliptic equation:

$$
\begin{equation*}
\frac{\partial^{2} \bar{\omega}_{x m}}{\partial \eta_{\mathrm{g}}^{2}}+\frac{1}{\eta_{\mathrm{g}}} \frac{\partial \bar{\omega}_{x m}}{\partial \eta_{\mathrm{g}}}+\frac{\partial^{2} \bar{\omega}_{x m}}{\partial \bar{x}^{2}}=0 \tag{5}
\end{equation*}
$$

Using Eqs. (1) and (3), we find

$$
\begin{equation*}
\bar{\omega}_{x m}=\frac{1}{\eta_{\omega}}\left(\frac{2 \eta_{\omega}}{1+\eta_{\omega}^{x}}\right)^{\frac{1+\eta_{\omega}^{\alpha}}{1-(x-1) \eta_{\omega}^{x}}} \simeq \frac{2}{1+\eta_{\omega}} \tag{6}
\end{equation*}
$$

(substitution error no greater than $4 \%$ ).
Considering the mutual independence of the variables $\eta_{g}$ and $\bar{x}$, we can obtain the general solution of Eq. (5) relative to $\eta_{\omega}$ by comparing two particular solutions $\left[\eta_{\omega}=\eta_{\omega}(\bar{x})\right.$, $\eta_{g}=0$; $\left.\eta_{\omega}=\eta_{\omega}\left(\eta_{g}\right), \partial \eta_{\omega} / \partial x=0\right]$ on the basis of the boundary conditions of the problem.

For chambers with a longitudinally distributed entry (see Fig. 1):

$$
\text { at } \begin{align*}
& \eta_{\mathrm{g}}=0 \text { and } \bar{x}=0 \quad \eta_{\omega}=\eta_{p} ; \\
& \text { at } \bar{x} \geqslant \bar{x}_{1 \mathrm{lim}} \quad \eta_{\omega}=0 ;  \tag{7}\\
& \text { at } \eta_{\mathrm{g}} \rightarrow 1 \quad \eta_{\omega} \rightarrow 1
\end{align*}
$$

As a result of the solution, the following theoretical expression was obtained:

$$
\begin{equation*}
\eta_{\omega, x}=\frac{2}{1+K_{x} \ln \eta_{\mathrm{g}}}-1 \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
K_{x}=\frac{1}{5}\left\{1-2\left[\left(1-\frac{X-X_{\mathrm{i}}}{1-X_{\mathrm{i}}}\right)\left(\frac{1-X_{\mathrm{i}}}{1+\eta_{p}}+X_{\mathrm{i}}\right)+\frac{X-X_{\mathrm{i}}}{1-X_{\mathrm{i}}}\right]\right\} \tag{9}
\end{equation*}
$$

is a coefficient considering the effect of the longitudinal coordinate on the distribution of $w_{\Phi}$ in the axial zone.

Equation (8) is valid in the range $\eta_{g}^{0} \leqslant \eta_{g} \leqslant 1$, where $\eta_{g}^{0}=6.74 \cdot 10^{-3} \leftrightarrows 0$ is the limiting minimum degree of loading, chosen with allowance for features of the solution and for its best agreement with the test data.

When $X_{i} \leqslant X \leqslant 1$, the calculation is performed by (8) and (9). When $1 \leqslant X \leqslant X_{k}$, it is performed by (8) with a value $\mathrm{K}_{\mathrm{X}}=\mathrm{K}_{1 \mathrm{im}}=$ const. The latter corresponds to a similitudinous distribution $\bar{w}=\bar{w}(\eta)$ along the chamber (the coordinate $X$ ).

For unloaded cyclone units, with $X_{i}=0$ Eq. (8) is simplified:

$$
\begin{equation*}
\eta_{\omega, x}=\frac{1}{\frac{1-X}{1+\eta_{p}}+X}-1 \tag{10}
\end{equation*}
$$

In cyclone chambers with a localized entry shifted toward the outlet hole, the boundary conditions are stated in a more complicated manner (Fig. 3a):

$$
\begin{align*}
\text { at } \quad \bar{x}=0 & \eta_{\omega}=\eta_{p} \\
\text { at } \bar{x}=\bar{x}_{\theta} & \eta_{\omega}=\eta_{\omega_{0}}  \tag{11}\\
\text { at } \bar{x} \geq \bar{x}_{\lim } & \eta_{\omega}=0
\end{align*}
$$

Solution of Eq. (5) together with conditions (11) leads to the following relation ( $\eta_{g}=0$ ): at $0 \leqslant x \leqslant X_{0}$

$$
\begin{equation*}
\eta_{\omega, x}=\frac{1}{\frac{1-X / X_{0}}{1+\eta_{p}}+\frac{X / X_{0}}{1+\eta_{\omega_{0}}}}-1 \tag{12}
\end{equation*}
$$

at $X_{0} \leqslant X \leqslant 1$

At $x \geqslant 1, \eta_{\omega, x}=0$.

$$
\begin{equation*}
\eta_{\omega a, x}=\frac{1+\eta_{\omega_{0}}}{1+\eta_{\omega_{0}} \frac{X-X_{0}}{1-X_{0}}}-1 \tag{13}
\end{equation*}
$$

Figure 3 shows a comparison of calculated and theoretical values of $\eta_{\omega, x}$ for the aboveexamined flow variants. In performing the calculations it was considered that, in accordance with the test data in [19, 22],

$$
\begin{align*}
& \bar{r}_{p} / \bar{r}_{\text {out }}=\eta_{p} / \eta_{\text {out }}=0.72-0.122 \bar{f}_{\mathrm{in}} \bar{f}_{\text {out }} \\
& \bar{x}_{\mathrm{iim}} / \eta_{p} \approx 8 ; \bar{x}_{0} / \eta_{p} \approx 1.85 ; \eta_{\omega_{0}} / \eta_{p} \approx 1,3 \tag{14}
\end{align*}
$$

The scale values of $w_{\Phi} m$ and $r_{\varphi m}$ were determined in accordance with the recommendations in [23, 24].

It is known that the radial distribution of static pressure in a cyclonic flow is determined mainly by the profile of $w_{\varphi}$. We can write the following in conformity with the equation of radial equilibrium of a rotary flow. Here, we follow [2, 22], with allowance for the proposed method of determining the exponent $n$ for the axial region

$$
\begin{equation*}
\bar{P}=-2 \int_{\eta=0 ; \eta g}^{\eta=1} \frac{1}{\eta}\left(\frac{2 \eta}{1+\eta^{x}}\right)^{-\frac{2\left(1+\eta_{\omega, z}^{x}\right)}{1-(x-1) \eta_{\omega, x}^{x}}} d \eta+C \tag{15}
\end{equation*}
$$

where $C$ is the constant of integration; $C=0$ when $\eta=\eta_{p}$ for unloaded chambers; $C=\bar{P}_{g}$ when $\eta=\eta_{g}$ for loaded chambers [6].

Figure 2 b shows empirical distributions of tangential and axial velocity and static pressure_in the plane of the outlet hole of a cyclone chamber ( $x=0$ ), as well as calculated curves for $\bar{w}$ and $\bar{P}$. It can be seen that in the region of negative values of $\bar{u}$, where the rotary and axial components of velocity become commensurate, and as $\eta \rightarrow 0 \bar{W} \rightarrow 0$, the effect of the law of radial equilibrium diminishes. This leads to stratification of curve 3 and the test data. Under these conditions, theoretical relation (15) can be roughly corrected to allow for the distribution of $u$ in the range $0 \leqslant \eta \leqslant \eta_{u}$ by using the Bernoulli equation for spiral flow. Then

$$
\begin{equation*}
\bar{P}=\bar{P}_{n=0}+\bar{u}^{2} \tag{16}
\end{equation*}
$$

To analyze the thermal problem, we will change over to a dimensionless form of Eq. (4), having taken $\bar{w}_{x m}, t_{0}$, and $\mathrm{dg}_{\mathrm{g}}$ as the scale quantities:

$$
\begin{equation*}
\frac{1}{\text { Ho }} \frac{\partial \bar{\omega}_{x}}{\partial \bar{t}}=\frac{1}{\operatorname{Re}_{\omega}} \nabla^{2} \bar{\omega}_{x} \tag{17}
\end{equation*}
$$

It can be seen from Eq. (17) that using the dimensionless equation reveals that there are two governing similitude numbers: homochronicity (Strouhal), which is excluded for the steadystate problem being examined, and the angular Reynolds number:

$$
\begin{equation*}
\operatorname{Re}_{\omega}=\frac{\omega_{x m} d_{\mathrm{g}}^{2}}{v} \tag{18}
\end{equation*}
$$



Fig. 3. Dependence of the dimensionless radius $\eta_{\omega}, x$ on the longitudinal coordinate and the degree of loading: a) unloaded chamber: 1) calculation by Eq. (10) ; 2) (12) ; 3) (13); tests: 4) distributed entry; 5) localized entry; b) loaded chamber: lines calculation by Eq. (8) ; points - experiment: 1) $X=0.6-0.8$; 2) $X \geqslant 1$.

The local Reynolds number, with allowance for Eqs. (6) and (8) for $\bar{\omega}_{x m}$ and $\eta_{\omega, x}$, will be represented as follows:

$$
\begin{equation*}
\mathrm{Re}_{\omega, x}=2 \eta_{g}\left(1+K_{x} \ln \eta_{g}\right) \mathrm{Re}_{\varphi m}=2 \eta_{\mathrm{g}} \Phi_{x} \mathrm{Re}_{\Phi m} \tag{19}
\end{equation*}
$$

The similitude equation for calculating the local heat-transfer coefficients (without allowance for the effect of direction of q) has the form

$$
\begin{equation*}
\mathrm{Nu}_{x}=A_{1} \eta_{\mathrm{g}}^{m_{1}} \Phi_{x}^{m_{2}} \mathrm{Re}_{q m}^{n} \tag{20}
\end{equation*}
$$

where $A_{1}, m_{1}, m_{2}$, and $n$ are constants determined by analyzing empirical data.
Similar to (20), in analyzing heat transfer in the theoretical expression for Nul averaged over the length of the surafce $\bar{\tau}_{g}=X_{k}-X_{i}$, we should introduce the mean integral value of the function $\Phi_{X}$, calculated accordingly from the mean value of $K_{x}$ :

$$
\begin{equation*}
K_{l}=\frac{1}{\bar{l}_{\mathrm{g}}} \int_{0}^{\bar{l}_{\mathrm{g}}} K_{x} d X \tag{21}
\end{equation*}
$$

Using Eq. (9), we obtain:
at $X_{i} \leqslant X_{k} \leqslant 1$

$$
\begin{equation*}
K_{l}=\frac{1}{5}\left[1+\left(\frac{\bar{l}_{\mathrm{g}}}{1-X_{\mathrm{i}}}-2\right)\left(\frac{1-X_{\mathrm{i}}}{1+\eta_{p}}+X_{\mathrm{i}}\right)-\frac{\bar{l}_{\mathrm{g}}}{1-X_{\mathrm{i}}}\right] \tag{22}
\end{equation*}
$$

at $\mathrm{X}_{\mathrm{k}} \geqslant 1$

$$
\begin{equation*}
K_{l}=\frac{1}{5}\left[\frac{\eta_{p}}{\eta_{p}+1} \frac{\left(1-X_{\mathrm{i}}\right)^{2}}{\bar{l}_{\mathrm{g}}}-1\right] . \tag{23}
\end{equation*}
$$

Analysis of the available test data on convective heat transfer [25, 26] shows a good possibility of it being generalized from common positions when the above assumptions are used.

It is of considerable interest to use integral methods of calculating the heat transfer of a cylinder in a twisted axisymmetric flow using data obtained earlier on the formation of the hydrodynamic and thermal boundary layers [27, 28]. The solution is based on the following integral relation for the thermal boundary layer:

$$
\begin{equation*}
\frac{d}{d r_{\mathrm{g}}} \int_{0}^{\delta_{\mathrm{t}} \cdot \mathrm{y}} w_{\phi} \hat{v} d y=\frac{q_{\mathrm{g}}}{\rho c_{p}} \tag{24}
\end{equation*}
$$

where $y=r-r_{g}$.
It was established experimentally that the boundary layer depends mainly on the coordinate $\eta_{g}$. In the first approximation we assume that the wall boundary layer is thin and that its transverse curvature can be ignored [28], and that the distributions of $w_{\varphi}$ and $\vartheta$ are described well by the power formulas

$$
\begin{align*}
& \frac{w_{p}}{w_{\mathrm{h}, \mathrm{y}}}=\left(\frac{y}{\delta_{\mathrm{h}, \mathrm{y}}}\right)^{m} ;  \tag{25}\\
& \frac{\vartheta}{\vartheta_{\mathrm{t} . \mathrm{y}}}=\left(\frac{y}{\delta_{\mathrm{t} . \mathrm{y}}}\right)^{m} . \tag{26}
\end{align*}
$$

Having determined the shear stress and heat flux within the turbulent boundary layer by the formulas

$$
\begin{align*}
& \tau_{\Phi \mathrm{g}}=\rho\left(v+\varepsilon_{\sigma}\right) d w_{\Phi} / d y  \tag{27}\\
& q_{\mathrm{g}}=\rho c_{p}\left(a+\varepsilon_{q}\right) d \boldsymbol{v} / d y \tag{28}
\end{align*}
$$

and having divided (28) by (27) with allowance for the fact that $\nu \ll \varepsilon_{\sigma}, a \ll \varepsilon_{\mathrm{q}}$ in the turbulent core, we obtain the relation

$$
\begin{equation*}
\frac{q_{\mathrm{g}}}{\tau_{\Phi \mathrm{g}} c_{p}}=\frac{1}{\operatorname{Pr}_{\mathrm{t}}}\left(\frac{\delta_{\mathrm{h} . \mathrm{y}}}{\delta_{\mathrm{t} . \mathrm{y}}}\right)^{m} \frac{\theta_{\mathrm{t} . \mathrm{y}}}{w_{\mathrm{Th} . \mathrm{y}}} \tag{29}
\end{equation*}
$$

The quantity $W_{\varphi} h . y$ in Eq. (29) is best expressed through the characteristic velocity in the core of the cyclonic flow - $\mathrm{w} \varphi \mathrm{m}$ - using the solution of the dynamic problem.

As in [27] the dimensionless thickness of the wall boundary layer $\bar{\delta}_{\mathrm{h} . \mathrm{y}}=\delta_{\mathrm{h} . \mathrm{y}} / \mathrm{dg}$ is assumed to be a function of $\operatorname{Re}_{\varphi m}$;

$$
\begin{equation*}
\bar{\delta}_{\mathrm{h} \cdot \mathrm{y}}=\frac{\delta^{+}}{\sqrt{\frac{A}{2}}} \operatorname{Re}_{\phi m}^{2 / 2-1} \tag{30}
\end{equation*}
$$

where $\delta^{+}$is the nominal thickness of the boundary layer; $A$ and $z$ are coefficients in the equation for calculating the dimensionless friction coefficient:

$$
\begin{equation*}
c_{f}=\frac{2 \tau_{\Phi \mathrm{g}}}{\rho e_{\Phi m}^{2}}=A \operatorname{Re}_{\Phi m}^{-z} \tag{31}
\end{equation*}
$$

With allowance for Eq. (30), the dimensionless radius of the external boundary of the hydrodynamic boundary layer

$$
\begin{equation*}
\eta_{h_{. y}}=\left(1+\frac{2 \delta^{+}}{\sqrt{\frac{A}{2}}} \operatorname{Re}_{\phi n i}^{2 / 2-1}\right) \eta_{\mathrm{g}} . \tag{32}
\end{equation*}
$$

Inserting Eqs. (3) and (32) into (1) yields

$$
\begin{equation*}
w_{\Phi \mathrm{h}, \mathrm{y}}=\left[\frac{2\left(1+2 \delta^{+} / \sqrt{A / 2} \mathrm{Re}_{\varphi m}^{2 / 2-1}\right)}{1+\left(1+2 \delta^{+} / \sqrt{A / 2} \mathrm{Re}_{\varphi m}^{2 / 2-1}\right.} \frac{\eta_{\mathrm{g}}}{)^{x} \eta_{\mathrm{g}}^{x}}\right]^{\frac{1+\eta_{\omega, x}^{x}}{1-(x-1) \eta_{\omega, x}^{x}}} w_{\varphi m} \tag{33}
\end{equation*}
$$

With the prescription of specific numerical values of $\delta^{+}, A, z$, and $x$ for the flow variants with so-called "free" and "compressed" maxima of $w_{\varphi} \cdot[27]$, $w_{\varphi} h . y$ is unambiguously determined by the values of $\eta_{g}$ and $X$ or $\bar{l}_{g}$ and $\operatorname{Re}_{\varphi m}$. Figure 4 a presents a graphic interpretation of (33) for flow on the similitudinous section ( $X \geqslant 1$ ) with a "compressed" maximum of $w$. To simplify further calculations, the awkward expression (33) can be approximated by the simpler power relation

$$
\begin{equation*}
w_{\varphi h_{\mathrm{o}} \mathrm{y}}=B \eta_{\mathrm{g}} \mathrm{Re}_{\varphi m}^{-k} w_{\varphi m}, \tag{34}
\end{equation*}
$$

where $B$ and $k$ are coefficients which in the general case depend on the friction law (31), $X$ or $\bar{\tau}_{g}$, and the range of $\operatorname{Re}_{\varphi m}$.


Fig. 4. Dependence of $\bar{W}$.y on $\operatorname{Re}_{\varphi m}$ and $\eta_{g}$ (a) and comparison of experimental data on the local (b) and mean (c, d) heat transfer of axisymmetric cylinders against calculated results: b) $1-\bar{d}_{\text {out }}=0.5$; $\operatorname{Re}_{\varphi \mathrm{m}}=1.32 \cdot 10^{5} ; 2-$ Gut $_{\text {out }}=0.6$; $\operatorname{Re}_{\text {¢m }}=1.96 \cdot 10^{5} ; 3-\bar{d}_{\text {out }}=0.7$;
$\mathrm{Re}_{\mathrm{pm}}=1.49 \cdot 10^{5} ; \mathrm{d}_{\mathrm{g}}=0.373$ [26];
c) 1 - calculation by Eq. (37); 2, 3-1imiting laws of heat transfer; d) 4 - calculation by Eq. (38).

Having calculated the definite integral in the left side of Eq. (24) with allowance for Eqs. (25), (26), and (34) and having performed certain transformations, we obtain a linear differential equation

$$
\begin{equation*}
\frac{d}{d \eta_{\mathrm{g}}}\left(\frac{\delta_{\mathrm{t} . \mathrm{y}}}{\delta_{\mathrm{h} . \mathrm{y}}}\right)^{2 m+1}+\frac{1}{\eta_{\mathrm{g}}}\left(\frac{\delta_{\mathrm{t} . \mathrm{y}}}{\delta_{\mathrm{h} . \mathrm{y}}}\right)^{2 m+1}=\frac{E}{\eta_{\mathrm{g}}^{3}} \mathrm{Re}_{\mathrm{pm}}^{2 k-3 / 2 z+1}, \tag{35}
\end{equation*}
$$

where

$$
E=\frac{2 m+1}{4} \frac{\sqrt{A^{3} / 2}}{B^{2} \delta^{+\mathrm{Pr}_{t}}}
$$

Solution of Eq. (35) makes it possible to find the relation between the thicknesses of the thermal and hydrodynamic boundary layers:

$$
\begin{equation*}
\frac{\delta \mathrm{t}_{\mathrm{t}} \mathrm{y}}{\delta_{\mathrm{h} . \mathrm{y}}}=\left[E\left(\frac{1}{C_{1} \eta_{\mathrm{g}}}-\frac{1}{\eta_{\mathrm{g}}^{2}}\right)\right]^{\frac{1}{2 m+1}} \operatorname{Re}_{\varphi m}^{\frac{2 k-3 / 2 z+1}{2 m+1}} \tag{36}
\end{equation*}
$$

Here, $C_{1}=n_{g}^{o}$ is the constant of integration.
Having inserted (31) and (36) into (29) and having isolated Nu in the right side, we obtain a formula to calculate the local or mean (over the length of the cylinder) heat transfer:

$$
\begin{equation*}
\mathrm{Nu}=\frac{1}{2} \frac{A}{B} \frac{\operatorname{Pr}}{\operatorname{Pr}_{\mathrm{t}}} \frac{1}{\eta_{\mathrm{g}}}\left[\left(\frac{1}{\eta_{\mathrm{g}}^{\theta} \eta_{\mathrm{g}}}-\frac{1}{\eta_{\mathrm{g}}^{2}}\right) \frac{2 m+1}{4} \frac{\sqrt{A^{3} / 2}}{B^{2} \delta^{+} \mathrm{Pr}_{\mathrm{t}}}\right]-\frac{m}{2 m+1} \operatorname{Re}_{\varphi m}^{n} \tag{37}
\end{equation*}
$$

where

$$
n=1+k-z-\frac{m}{2 m+1}\left(2 k-\frac{3}{2} z+1\right)
$$

Equation (37) was compared (Fig. 4 b and c ) with experimental data on local and mean values of the numbers Nu. For the variant with a "compressed" maximum of $w_{\varphi}$ (Fig. 4c), the comparison was made with $\mathrm{A}=0.234 ; \mathrm{m}=0.1 ; z=0.4 ; \delta^{+}=270 ; \operatorname{Pr}=0.71 ; \operatorname{Pr}_{\mathrm{t}}=1$. The value of $\mathrm{Pr}_{\mathrm{t}}$ evidently changes over the thickness of the boundary layer in the general case and depends on the degree of turbulence of the external (stream) flow. Thus, for the variant with the "free" maximum of $w_{\epsilon}$, the best agreement between the theoretical and empirical data is obtained at $\operatorname{Pr}_{\mathrm{t}}=0.75-0.8$.

It follows from Fig. $4 b$ that the calculation of local values of Nu satisfactorily agrees with the experimental values in the core of the flow.

The deviation seen in the end zones (at $\mathcal{Z}_{\mathrm{g}}=\mathrm{L}$ ) is due to specific features of the flow [26]. The dashed curves 2 and 3 in Fig. 4c correspond to the two limiting laws of heat transfer obtained from Eq. (37): 2 - with decay of the stream boundary layer (at $\mathrm{R}_{\boldsymbol{q} \mathrm{m}} \leqslant \operatorname{Re}_{\Phi \mathrm{m}}^{\mathrm{br}} \approx 6$. $10^{4} \mathrm{w}_{\mathrm{\Phi} h . \mathrm{y}}=\mathrm{w}_{\mathrm{q} \mathrm{m}}$ ); 3 - with decay of the wall boundary layer (at $\operatorname{Re} \varphi \mathrm{m} \rightarrow \infty \delta_{\mathrm{h}, \mathrm{y}} \rightarrow 0, \mathrm{c}_{\mathrm{f}} \rightarrow 0$ ).

The effect of the degree of loading $\eta_{g}$ on the mean heat transfer can be analyzed (Fig. 4 ), having represented (37) in dimensionless form:

$$
\begin{equation*}
\overline{\mathrm{Ko}}_{l}=\frac{\mathrm{Nu}_{l} / \mathrm{Re}_{\phi m}^{n}}{\left(\mathrm{Nu}_{l}\right)_{\eta_{\mathrm{g}}=1} / \mathrm{Re}_{\mathrm{q} m}^{n}}=\frac{1}{\eta_{\mathrm{g}}}\left(\frac{\frac{1}{\eta_{\mathrm{g}}^{0} \eta_{\mathrm{g}}}-\frac{1}{\eta_{\mathrm{g}}^{2}}}{\frac{1}{\eta_{\mathrm{g}}^{0}}-1}\right)^{-\frac{m}{2 m+1}} \tag{38}
\end{equation*}
$$

It follows from Eq. (38) that with a decrease in $\eta_{g}$ heat transfer increases: with a decrease in $\eta_{g}$ from 0.8 to $0.6, \bar{K}_{\ell} \mathcal{L}$ increases by a factor of 1.3 .

The standard deviation of the empirical points from the theoretical curves is no greater than $\pm 8-12 \%$.

## NOTATION

$R, D, L$, radius, diameter, and length of the working volume of the cyclone chamber; $r, x$, running radius and longitudinal coordinate; $\delta$, thickness of the boundary layer; $\eta_{-}=r / r_{\varphi m}$; $\bar{x}=x / r_{4}, X=\bar{x} / \bar{x}_{\text {lim }}$, dimensionless running radius and longitudinal coordinate; $\bar{d}=d / D$, dimensionless diameter; $\overline{\mathrm{f}}=4 \mathrm{f} / \pi \mathrm{D}^{2}$, dimensionless area; V , $\omega$, total linear and angular velocity of the flow; $w_{q}, w_{x}$, tangential component of the linear and axial component of the angular velocity of rotary flow; $\bar{w}, \bar{u}, \bar{\omega}_{X}$, dimensionless (referred to $\omega_{\Phi_{m}}, \omega_{\mathrm{X} \eta}=1$ ) tangential and axial components of linear velocity and axial component of angular velocity; $t$, time; $T$, temperature; $v=T_{g}-T$, excess temperature; $\overline{\mathrm{P}}=2 \mathrm{P}_{\mathrm{c}} / \rho \mathrm{w}_{\mathrm{qm}}^{2}$, dimensionless excess static pressure; $\overrightarrow{\mathrm{r}}_{\mathrm{p}}=$ $r_{p} / R, \eta_{p}, \eta_{u}$, dimensionless radii of zero excess static pressure and axial velocity component; $q$, heat flux; $\rho, c_{p}, v, a$, density, isobaric specific heat, kinematic viscosity, and diffusivity; $\varepsilon_{\sigma}, \varepsilon_{q}$, turbulent analogs of $v, a ; \operatorname{Pr}, \operatorname{Pr}_{t}$, molecular Prandtl number and its turbulent analog; $\tau_{\varphi}$, tangential component of the shear stress; $\operatorname{Re}_{q} m=w_{q} d g / \nu, N u=\alpha d_{g} / \lambda$, Reynolds and Nusselt numbers. Indices: g, loading, value on the cylinder surface; in, out, inlet and outlet values; $m$, $w$, values pertaining to the maximum value of the components of linear and angular velocity; $c$, value on the boundary of the core of the flow; i, initial; $k$, final; lim, limiting; $x$, local; $l$, mean over the length; 0 , nominal, zero; h.y, t.y, values on the boundaries of the hydrodynamic and thermal boundary layers; br, boundary; a bar above a quantity means that it is dimensionless, while a prime means that it is pulsative; a quantity in angular brackets is an average.

## LITERATURE CITED

1. L. A. Vulis and B. P. Ustimenko, "Aerodynamics of a cyclone furnace chamber," Teploenergetika, No. 9, 3-10 (1954).
2. A. N. Shtym and P. M. Mikhailov, "Aerodynamics of a twisted flow in cyclone-vortex chambers," Izv. Vyssh. Uchebn. Zaved., Energ., No. 11, 50-53 (1965).
3. I. I. Smul'skii and V. I. Kislykh, "Study of velocity and pressure fields in a vortex chamber," in: Studies of Hydrodynamics and Heat Exchange [in Russian], Inst. Teplofiz, Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1976), pp. 200-206.
4. S. V. Bezuglyi and A. P. Fursov, "Numerical study of flow hydrodynamics in the axial zone of a semiinfinite vortex," in: Mathematical Methods of Analyzing Dynamic Systems [in Russian], Vol. 3, Kharkov Aviation Institute (1979), pp. 14-19.
5. T. I. Zelenyak, V. I. Kislykh, and 0. G. Provorova, "Model of gas dynamics in the axial zone of vortex apparatus," in: Continuum Dynamics, No. 46, Inst. Teplofiz. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1980), Pp. 33-45.
6. E. N. Saburov, Aerodynamics and Convective Heat Transfer in Cyclone Heaters [in Russian], Leningrad State Univ. (1982).
7. A. D. Al'tshul' and M. Sh. Margolin, "Generalized formula for the distribution of Peripheral velocity in vortices," Inzh.-Fiz. Zh., 18, No. 4, 731-733 (1970).
8. K. V. Grishanin, "Dynamics of bottom flow in a stationary whirlpool," in: Inland Waterways [in Russian], No. 61, Leningr. Inst. Vodn. Transporta, Moscow (1964), pp. 36-43.
9. A. A. Khalatov, "Calculation of the rotational velocity profile in a cylindrical channel with a twisted flow at the inlet," Promyshlennaya Teplotekh., 1, No. 2, 75-78 (1979).
10. A. V. Tonkonogii, E. Balfanbaev, and P. V. Kopytkov, "Aerodynamics of cyclone chambers with a top gas outlet," in: Problems of Heat Engineering and Applied Thermophysics [in Russian], Vo1. 4, Nauka KazSSR, Alma-Ata (1967), pp. 90-108.
11. S. V. Karpov and E. N. Saburov, "Scheme for calculating the aerodynamics of cyclone chambers with peripheral gas outlet," in: Tenth All-Union Scientific-Technical Conference on Cyclone Power-Engineering Processes (Summary of Documents), VNIPIenergoprom, Moscow (1978), pp. 41-42.
12. B. P. Ustimenko, Turbulent Transport in Rotating Flows [in Russian], Nauka, KazSSR, AlmaAta (1977).
13. T. K. Luk'yanovich, "Investigation of the aerodynamics of the peripheral zone of cyclonevortex chambers," Candidate's Dissertation, Engineering Sciences, Leningrad (1975).
14. H. O. Anwar, "Flow in a free vortex," Water Power, April, 153-161 (1965).
15. E. R. Hoffmann and P. N. Joubert, "Turbulent line vortices," J. Fluid Mech., 16, Pt. 3, 395-411 (1969).
16. G. M. Yakubov, "Generalization of the aerodynamic characteristics of cyclone chambers," Izv. Akad. Nauk KazSSR, 1, No. 12, 105-118 (1957).
17. B. P. Ustimenko and M. A. Bukhman, "Study of averaged and pulsative characteristics of a flow in cyclone chambers," in: Problems of Heat Engineering and Applied Thermophysics [in Russian], Vol. 5, Nauka KazSSR, Alma-Ata (1969), pp. 95-105.
18. Murakami Mitsukiya, Katayama Yutaka, Jida Yoshihiko, and Kito Osami, "Experimental study of a twisted flow in pipes," Nippon Kikai Gakkai Rombunshu, Trans. Jpn. Soc. Mech. Eng., 41, No. 346, 1793-1801 (1975).
19. E. N. Saburov and S. V. Karpov, "Resistance of cyclone chambers in the nonsimilitudinous region of stream flow," Inzh.-Fiz. Zh., 28, No. 2, 354-355 (1975).
20. E. V. Volkov, "Rotation of a gas in the axial zone of a cyclone chamber," Inzh.-Fiz. Zh., 3, No. 8, 26-30 (1960).
21. A. M. Lyatkher, Turbulence in Hydraulic Engineering Works [in Russian], Énergiya, Moscow (1968), pp. 38-40.
22. A. N. Shtym and A. S. Latkin, "Zero level of static pressure in cyclone-vortex chambers," Inzh.-Fiz. Zh., 27, No. 3, 532-533 (1974).
23. S. V. Karpov and E. N. Saburov, "Method of calculating the aerodynamic characteristics of cyclone chambers," Khim. Neft. Mashinostr., No. 7, 20-22 (1977).
24. E. N. Saburov and S. V. Karpov, "Method of calculating the aerodynamics of cyclone-vortex heaters," Izv. Vyssh. Uchebn. Zaved., Energ., No. 8, 71-77 (1975).
25. E. N. Saburov and S. V. Karpov, "Experimental study of heat exchange of a cylinder with a stabilized twisted flow," Izv. Akad. Nauk SSSR, Energ. Transp., No. 3, 166-169 (1976).
26. E. N. Saburov and S. I. Ostashev, "Study of heat exchange of a cylindrical insert coaxial with the working volume of a cyclone chamber," Izv. Vyssh. Uchebn. Zaved., Energ., No. 6, 66-72 (1979).
27. E. N. Saburov, S. V. Karpov, Yu. L. Leukhin, and S. I. Ostashev, "Study of the boundary layer on the surface of a cylinder in a cyclonic flow," Izv. Vyssh. Uchebn. Zaved., Energ., No. 6, 86-93 (1977).
28. É. N. Saburov, S. V. Karpov, Yu. L. Leukhin, and S. I. Ostashev, "Calculation of heat exchange of a cylinder with a cyclonic flow located coaxially with respect to the cylinder," Izv. Vyssh. Uchebn. Zaved., Energ., No. 10, 102-107 (1977).
